How Tempo Distortions Affect the Comparative Analysis of Mortality: The Case of Diverging Mortality Between Eastern and Western Europe

or

The Importance of Mortality Tempo Adjustment: Theoretical and Empirical Considerations

- provisional paper (references not yet updated), please do not cite or quote -

Marc Luy

University of Rostock, Institute for Sociology and Demography

E-Mail: email@marc-luy.de

Abstract

This paper presents both, a support of Bongaarts and Feeney's tempo approach in mortality and its first empirical application. In the first part it is shown that the common critics on the idea of tempo distortions in life expectancy are not justified: if we accept the need for tempo-adjustment in the period TFR we have equally to accept the need for tempo-adjustment in period life expectancy. While tempo-adjustments of the TFR lead mainly to somewhat higher estimates for the hypothetical family size under current fertility conditions this paper shows that tempo-adjustment of life expectancy can provide a completely different picture of current mortality conditions as compared to conventional life expectancy. Applying this method for analysing the mortality gap between Western and Eastern Germany provides remarkable results: the differences in survival conditions between West and East Germany are still considerably higher than generally expected and the survival gap between West and East Germany started to close some years later than trends in conventional life expectancy suggest. Since life expectancy without any adjustment for tempo effects is one of the most used demographic tools in order to analyze mortality it can be concluded that we probably have to change much more of what we thought to know about mortality trends and their driving factors. On the base of these findings the paper will be extended to estimates for tempo-adjusted life expectancy in several Eastern and Western European countries and thus the East-West mortality gap in Europe.

Introduction

One of the main goals of quantitative demography is the derivation of period measures with a clear and distinct meaning in order to analyzing both, demographic developments in time and current demographic conditions in different populations. For more than a century demographers thought to know how to calculate and interpret such period measures like the total fertility rate (TFR) or the life expectancy at birth (e_0). Both are summary measures representing current fertility respective mortality conditions standardized for the actual age composition of populations that drives the number of observed events and thus the values of crude rates. However, with a series of papers Bongaarts and Feeney (1998, 2002, 2003, 2005) recently claimed that such summary measures should not only be standardized for age but also for tempo effects that arise whenever demographic conditions are changing. Introducing this idea with corresponding formulas for tempo-adjustment Bongaarts and Feeney stirred the world of demographers and divided their community into the groups of tempo supporters and tempo opponents. Whilst a continuously growing number of scholars follow the tempo approach in fertility the tempo approach in mortality is still generally rejected. This is principally irrational since the basic idea behind the tempo approach is absolutely independent from the kind of demographic event. Moreover, it seems that tempo effects affect the current period measures for mortality to a considerable higher extent than they do with fertility measures. In the actual discussion about mortality tempo this aspect is completely missed since all existing papers deal solely with theoretical and technical questions, empirical applications are missing. One very interesting although not explicitly mentioned aspect of the initial mortality tempo paper of Bongaarts and Feeney (2002) is the fact, that the variance in life expectancy between the US, Sweden, Japan and France decrease from 3.4 years according to conventional life expectancy to only 1.7 years according to tempo-adjusted life expectancy. Applying the Bongaarts and Feeney method to the case of mortality differences between Eastern and Western Germany I will show that adjusting period life expectancy for tempo effects results in a considerable change the common knowledge. Before I try to explain the idea of tempo effects on a verbal base making clear why they actually do affect period life tables and why they should in fact be seen as distortions what is doubted so far by several scholars.

How mortality tempo affects period life expectancy

In principle, Bongaarts and Feeney's idea of tempo distortions is very simple and completely unrelated to the complex discussion it caused, especially connected with its application in the field of mortality. The explanation of this may be that mortality tempo effects have implications that seem to be in variance with established ways of modelling and analyzing mortality. The misunderstanding and confusion regarding the nature of tempo effects in period life expectancy seem to develop from four different roots:

1. Demographers usually develop their formulas in continuous time. However, tempo effects originate exclusively from discrete period rates and are then transferred to all demographic measures derived from such rates. In order to understand Bongaarts and Feeney's approach of tempo effects in period life expectancy this becomes a confusing problem since the circumstances causing tempo effects do not exist in the continuous force of mortality and thus cannot be identified in a continuous mortality model.
2. Bongaarts and Feeney's paper on mortality tempo is based on their paper "On the quantum and tempo of fertility". This causes some irritations since several scholars try to find a quantum effect in mortality that by definition cannot exist. It is however important to note that tempo effects are not necessarily connected to quantum. Tempo generally affects period rates and quantum is only affected by tempo effects if the period rates are used to estimate demographic quantum like it is done with the TFR. If period rates are used to derive any other demographic measures then these measures are affected by tempo distortions. It plays no role if they contain a quantum component or not like is the case in period life expectancy.
3. There seems to be the misunderstanding that Bongaarts and Feeney try to estimate a period measure with a certain cohort meaning. This is, however, not exactly the case. In their basic idea they rather use cohort experiences in order to estimate current changes in age-specific mortality conditions. The origin of this misunderstanding may possibly be found in the title of their original paper "How long do we live?" since any "we" does not exist in the logic of pure period measures and does only make sense in the perspective of cohorts. Another reason is probably the similarity between Bongaarts and Feeney's tempo-adjusted life expectancy and other period measures with clear defined cohort components, like the cross-sectional average length of life (CAL). However, something like a cohort projection is not intended by Bongaarts and Feeney.

4. Finally, scholars analyzing Bongaarts and Feeney's mortality tempo papers are usually mixing up two different questions: "do tempo effects distort period life expectancy?" and "do Bongaarts and Feeney's adjustment formulas provide adequate measures for adjusting tempo effects in the case of mortality?". Both questions are important but they have to be separated in order to do justice to Bongaarts and Feeney's tempo approach.

In order to avoid these misunderstandings in this chapter Bongaarts and Feeney's approach of tempo distortions in period life expectancy is derived on a more verbal base. The idea of mortality tempo effects derives directly from the idea of fertility tempo effects, which are known since more than half a century and which are widely accepted. In this connection Bongaarts and Feeney (1998) proposed a new method for estimating the tempo bias in period fertility rates and provided a formula to adjust the TFR for these distortions. Despite some critics, this formula made its way successfully into demographic research as can be seen in a continuously increasing number of papers applying this formula or confirming its effectiveness (e.g. Kohler and Philipov, 2001; Philipov and Kohler, 2001; Zeng Yi and Land, 2001, 2002; Sobotka 2004). As already mentioned, Bongaarts and Feeney's successive work on mortality tempo effects is, however, far away from broad acceptance. The most frequent critics against the Bongaarts/Feeney-approach to adjust life expectancy are based on the argument that life expectancy itself is a pure tempo measure and thus cannot be adjusted for tempo effects (Guillot, 2003b; Rodríguez, 2005; Wilmoth, 2005). This is, however, not the claim of Bongaarts and Feeney who focus their idea on tempo distortions in age-specific death rates, i.e. the base of life table techniques, and not on the measures that are derived from them.

In order to demonstrate the idea of tempo effects in mortality I use a simple illustration given by Feeney (2003) in his unpublished paper with the apt title "mortality tempo: a guide for the sceptic". Consider a population in which all births occur intermittently at 0.2 year intervals and in which all deaths occurring during some base year occur at exactly the midpoint of a single year of age. Suppose that, at the end of this base year, age at death within a certain age group begins to increase linearly at the rate of 0.2 years per year for all persons, and ceases increasing at the end of the year. This situation is graphically shown in the Lexis diagram in figure 1 for age 62, where the life lines of each cohort are traditionally represented by an arrow moving through time and age. In the base year t0 all deaths in age 62 occur exactly at age 62.5. During year t1 the age at death increases linearly with the given annual rate from 62.5 to

62.7. This level is reached in year t2 and remains constant from then on. Assume further, that annual numbers of births to the population have been constant and that the proportion of deaths in any birth cohort that occur at the i-th age at death is constant over all cohorts (that means unchanged mortality conditions until the base year t0). These two assumptions imply that each dot in figure 1 represents the same number of deaths and that each arrow represents the same number of persons surviving until age 62.5.

The decisive aspect is what happens with the number of deaths in year t1, the year of changing mortality. The five cohorts in t1 reaching age 62.5, the exact age at which those who do not survive the given age group die according to the old conditions, are marked with the letters A to E. Thus, cohort A is the oldest cohort reaching age 62.5 in year t1 and cohort E the youngest. Due to the assumed changes in mortality conditions during year t1, the ages at death of cohorts A to E increase steadily and cohort E is the first to reach the new age at death level of 62.7 years. Since each of these five cohorts lives somewhat longer than the preceding cohort, the intervals between deaths are longer than the intervals between births (before the year of changing mortality conditions t1 these intervals are identical). As a consequence, the deaths to the five cohorts that reach age 62.5 during year t1 are spread over a period longer than one year. This leads to the effect that the deaths of persons belonging to cohort E are shifted to year t2 as shown by the thick grey arrow in figure 1. The number of deaths in year t1 declines by 20 percent as compared to the situation before mortality conditions changed. This can clearly be seen in figure 1, where only four black dots are lying in year t1. If mortality would not have changed during that year, there would be five dots in year t1 as is demonstrated with the unfilled dots representing the age at death to cohorts according to the old mortality conditions until year t0. Figure 1 shows that this decline in the annual number of deaths is transitory in the sense that it disappears when age at death stops rising. From year t2 on, the intervals between births and deaths are identical again and there are five dots in each subsequent calendar year.

It is clear that the decrease in the number of deaths in year t1 leads to a decrease in the age-specific death rate. The decisive argument making an adjustment for tempo effects necessary is that the period death rate for year t1 will be lower than the period death rate for year t2, although the age at death is higher in t2 than it is in t1. Let N_x denote the number of persons reaching exact age x during each of the three years shown in figure 1, and let D_x denote the annual number of deaths during years t0 and t2 (the years with five "dots" of deaths). As already described, the number of deaths in year t1 is 20 percent less than D_x. Person years lived at age x in completed years for years t0 to t2 are $N_x - 0.5D_x$, $N_x - 0.4D_x$, and $N_x - 0.3D_x$. Consequently, the age-specific death rates for age $x = 62$ for the three years are

$$M_{x,t0} = \frac{D_x}{N_x - 0.5D_x},\ M_{x,t1} = \frac{0.8D_x}{N_x - 0.4D_x},\ \text{and}\ M_{x,t2} = \frac{D_x}{N_x - 0.3D_x}.$$

The subtracted terms in the denominators represent person years not lived by the persons who die at age x in completed years. Because of the rise in age at death, this term declines each year. The quantities for the three age-specific death rates can be expressed as

$$M_{x,t0} = \frac{1}{1/q_x - 0.5},\ M_{x,t1} = \frac{0.8}{1/q_x - 0.4},\ \text{and}\ M_{x,t2} = \frac{1}{1/q_x - 0.3},$$

where q_x denotes the probability of dying in age x, thus D_x / N_x. In a population with high life expectancy at birth, q_x-values for young and middle adult ages will be close to 0.01 or 0.001, the value of $1/q_x$ will be 100 to 1,000, and the impact of the subtractions in the denominators will be negligible. This shows that, for the hypothesized model population, age-specific death rates for all but the oldest ages and infancy will decline by approximately 20 percent between year t0 and year t1 and rise by approximately 25 percent between year t1 and year t2. According to Bongaarts and Feeney's approach, this temporary decline in age-specific death rates is a tempo effect. It results from the fact, that in situations of increasing age at death the number of deaths in the enumerator of the death rates declines by a considerable higher relative extent than do the risk years lived in the denominator of the death rates. The logic of this argument is neither limited to the simple assumptions of this example (constant number of births, birth intervals of 0.2 years) nor to one single age group. If we would increase the number of age groups and assume that ages at death in these age groups increase at different rates, different numbers of deaths will be shifted and the magnitude of the tempo effect will vary from one age group to another (for deeper descriptions see Feeney, 2003).

Since conventional life expectancy at birth is based on age-specific death rates calculated from annual number of deaths and risk years lived there should be no doubt that this measure might be considerably affected by such tempo effects. The most illuminating paper about the consequences of such biases on the interpretation of period data was written by Vaupel (2002) who called for a distinction between "life expectancy at current rates" and "life expectancy at current conditions".[1] The origin of this decisive difference can be found in the basic assumption of stable (or even stationary) conditions which is underlying all demographic period measures. The papers of Bongaarts and Feeney show that these assumptions do not only influence the possible interpretation of period measures, what we were already aware of. Much more important, the conventional way of calculating period measures could lead to a completely wrong message whenever the assumptions of stability respective stationary are not satisfied in the analysed year or period. In the case of life expectancy one can in fact generalize: if mortality declines, life expectancy will overestimate current conditions and if mortality is rising, life expectancy underestimates current survival conditions. This bias is the more marked the more intensive the changes are during the observed period.

All demographic period measures are hypothetical estimates in order to standardize for current demographic conditions. Since different populations experience changes in mean age at death differently, equivalent to changes in mean age at childbirth, tempo effects affect them differently like do different age compositions of the populations. Thus, tempo effects must generally be seen and treated as a distortion of period measures like the effects of population age composition. Bearing in mind the simple example presented above, where the usual death rate for year t1 provides a much lower value than actual mortality conditions should produce raises the question what meaning at all period measures based on current rates do have in a world of continuous demographic changes. This holds especially when populations with completely different demographic developments are compared, like the populations of West and East Germany. To assure that conventional period measures do not lead into a wrong direction it is necessary to look at tempo-adjusted measures, regardless if fertility, mortality, or any other demographic process is analysed.

Why Mortality differences between Western and Eastern Germany call for tempo-adjustment

The demographic changes and developments in Eastern and Western Germany are generally seen to offer a unique possibility to understand the interaction between societal, social respective economic conditions and population processes. Thus, the German experience is used in order to understand the causes of recent changes in mortality. Almost identical demographic composition and behaviour until 1945 were followed by 45 years of life under different political and socio-economic structures resulting in completely different demographic developments (Dinkel, 1992, 1994, 1999; Gjonca et al., 2000). With Reunification in 1990 the population in Eastern Germany returned to the Western societal and economic system what caused sudden changes in all its demographic processes. These special preconditions – leading some scholars to describe the Eastern German population as a kind of “natural experiment” (Dinkel, 1999; Vaupel et al., 2003) – generated a large number of researches about changes in Eastern German demography. In the field of mortality research especially the rapid convergence of survival conditions since 1990 following roughly two decades of continuous divergence are subject of central interest. The fact that both, the former increase and the recent decrease of the life expectancy gap between West and East Germany were mainly caused by age groups between 60 and 80 led to the central message that “it’s never too late” for increasing length of life (Vaupel et al., 2003).

Figure 2 shows the trends in period life expectancy at birth e_0 using standard life table techniques for West and East German women and men for each single calendar year from 1950 to 2000. The life table calculations are based on official population statistics, i.e. numbers of the living population and deaths for each calendar year and single age groups (see description in Luy 2004a). Regarding mortality differences between West and East Germany the presented time span can be subdivided into four central phases:

- The first phase from 1950 to roughly 1960 is characterized by irregular fluctuations with several years of mortality crossing over. These trends correspond with the waves of influenza that circulated in different years in East and West Germany (Luy 2004a). Thus, during these years no mortality differences can be stated between the two Germanys, neither for men nor for women.
- In the second phase approximately covering the period 1960 to 1975 the development of life expectancy in West and East Germany became more regularly, with a slightly higher mortality of East German women but a significantly lower mortality of East German men as compared to their West German counterparts. The differences in favour of East German men increased until the first half of the 1970s and reached a maximum of roughly one year in life expectancy at birth. However, this disadvantage of West German men is mainly affected by different definitions of live birth in East and West Germany which caused lower infant mortality rates in the former GDR for purely statistical reasons.[2] An analysis of age-specific differences between West and

East German mortality clearly shows that the higher life expectancy of East German men in these years was mainly (but not only) resulting from lower infant mortality (Luy 2004a).

- The third phase of life expectancy trends in West and East Germany starting in the middle of the 1970s is characterized for both sexes by a continuous divergence in the development of survival conditions in favour of West Germany. This development corresponds to the general divergence in mortality trends between all Western and Eastern European countries (see e.g. Caselli and Egidi, 1980; Bourgois-Pichat, 1985; Bobak and Marmot, 1996a, 1996b; Hertzman et al., 1996; Meslé and Hertrich, 1997; Vallin and Meslé, 2001; Meslé and Vallin, 2002). Figure 2 shows that this opening of the survival gap was caused by the fact that East German life expectancy at birth increased with a lower gradient for both sexes, while life expectancy in West Germany rose more rapidly (Höhn and Pollard, 1990, 1991; Scholz, 1996; Gjonça et al., 2000; Nolte et al. 2000a). The differences peaked in 1988 for women (almost 3 years) and in 1990 for men (roughly 3.5 years).
- These peaks – virtually concurring with German Reunification – were followed by a continuous closing of the gap in West-East German mortality differences until 2000, where the difference in e_0 was about 0.5 years for women and about 1.5 years for men. As can be seen in figure 2, with the beginning of this phase the differences in life expectancy trends between West and East Germany reversed as compared to the trends in the third phase. The now observable convergence of mortality levels is due to the fact that life expectancy rises faster in Eastern Germany since the beginning of the 1990s as compared to the West.

Figure 2 additionally visualizes the striking increase in mortality of East German men in 1990 that has been described as East German “mortality crises” (Dorbritz and Gärtner, 1995; Riphan, 1999; Nolte et al., 2000a, 2000b) and as one characteristic of a “demographic shock” East Germany’s in connection with the changes caused by Reunification (Eberstadt, 1994). However, long-term trends in survivorship completely contradict to such a description and rather call for an explanation of the rapid closing of the gap. The decisive question is: which factor or which factors could be responsible for the reversing of trends in mortality differences between West and East Germany within only one or two years of time?

The most factors discussed are the same that are assumed to be responsible for the general mortality gap between Western and Eastern European countries (e.g. Bobak and Marmot, 1996a, 1996b; Hertzman et al., 1996) making the search for the causes of mortality trends in East Germany to a subject of important interest exceeding the borders of Germany. It seems to be clear that finding the decisive cause(s) for the mortality differences between Eastern and Western Germany will be a decisive step in gaining a better understanding of general mortality differentials and especially of mortality differences between Western and Eastern Europe. A huge and continuously increasing number of studies follow this path based on the trends in live expectancy as shown in figure 2 (e.g. Chruscz, 1992; Dinkel, 1994, 1999; Schott et al., 1994; Becker and Boyle, 1997; Gjonça et al., 2000; Bucher, 2002; Nolte et al., 2002; Luy, 2004a, 2004b; Mai, 2004). However, following Bongaarts and Feeney's tempo approach we have to conclude that period life expectancy based on annual death rates is an imperfect solution for reflecting period mortality conditions, because period life expectancy calculated in the standard way is distorted whenever it is changing. In the previous chapter was shown that when the mean age at death rises (mortality decline), the death rates are biased downward and when the mean age at death falls (mortality increase), they are biased upward. Both decisive changes in the development of mortality differences between West and East Germany coincided with considerably different trends in life expectancy in the two parts of Germany: in phase 3, life expectancy increased successively in West Germany while it increased only slightly or remained constant in the East and in phase 4, life expectancy rose with a considerably higher pace in Eastern Germany as compared to the West. If these different trends in West and East Germany are causing tempo distortions in the sense of the Bongaarts and Feeney-approach, all researches on causes of East German excess mortality might be based on data leading to a distorted picture of mortality conditions in West and Germany and thus to the differences between them.

Methods for estimating tempo-adjusted life expectancy

In order to estimate tempo-adjusted life expectancy for West and East Germany I follow the approach of Bongaarts and Feeney (2002), who defined the tempo effect $S(t)$ in life expectancy in a year t as the absolute difference between the observed life expectancy at birth $e_0(t)$ and the tempo-adjusted life expectancy at birth $e_0^*(t)$ (by Bongaarts and Feeney called "average age at death"), thus

$$S(t) = e_0(t) - e_0^*(t)\,. \tag{1}$$

The measure $e_0^*(t)$ is defined as the average age at death in a population with constant number of births. This measure is closely related to the "cross-sectional average length of life" (CAL) as introduced by Brouard (1986) and Guillot (2003a) but not being absolutely identical (see Guillot 2003b). In a subsequent paper, Bongaarts and Feeney (2003) presented three different possibilities to similarly estimate tempo-adjusted period life expectancy from complete cohort data about births, deaths, and migration in order to empirically reconstruct a constant birth population for a certain period. However, for the West and East German populations such detailed data does not exist. In such a case where cohort data about births, numbers of death and migration cannot be used (at least not for a time span long enough), $e_0^*(t)$ can be estimated by solving the equation

$$e_0(t) = e_0^*(t) - \frac{1}{b}\ln\left(1 - \frac{de_0^*(t)}{dt}\right) \tag{2}$$

for $e_0^*(t)$ from conventional life table estimates, based on the assumptions that mortality under age 30 can be neglected and that the annual changes in the force of mortality follow a shifting Gompertz function.[3] For the detailed derivation of this formula see Bongaarts and Feeney (2002). As proposed by the authors the value b is estimated by fitting a Gompertz model to single-year age-specific death rates for ages 30-90.[4] Although cohort experiences are generally connected with age-specific period death rates and thus with the estimates of the Gompertz parameter b, equation (2) does not contain a direct cohort component and includes only elements derived from period data.

Table 1 presents the estimates of the parameter μ_0(t) and the average of the parameter b for the analysed populations from 1975 to 2000. The estimates for b for the whole series of single observation years can be found in appendix (c) and (d) of this paper. Corresponding to the observed death rates, μ_0(t) declines over time for all four populations. Similar to what is known for several other countries, the estimated values of b are close to 0.11 among males and 0.12 among females for both, West and East Germany. The annual estimates of b vary only little over time in each of these populations during the observation period, as can be seen from the standard deviation of b in table 1 or from the single values in appendix (c) and (d).

Like in the case of the populations analysed by Bongaarts and Feeney (2002), the Gompertz model fits the observed adult death rates very well, with the average variance explained (R^2) being more than 99 percent. On the base of these data, I used a three-step procedure similar to the procedure proposed by Bongaarts and Feeney (2002). First, I calculated annual estimates of $e_0(t)$ from 1950 to 2000 (enumerated from year 0 to year 50) with life tables in which mortality under age 30 was set to 0. Next, I smoothed these estimates by fitting a sixth degree polynomial using the computer program R. The resulting values for the smoothed time series for life expectancy $e_0(t)^S$ are given in appendix (c) and (d), and in figure 3 the corresponding functions are shown together with the original estimates for $e_0(t)$ with no mortality under age 30. It can be seen that the trends in $e_{30} + 30$ (what corresponds to setting mortality below age 30 to 0) are very similar to the trends in e_0 shown in figure 2. Only in the years 1950 to 1970 the trends differ slightly as a consequence of the fact that mortality below age 30 (especially infant mortality) had a higher impact on overall life expectancy than it has in more recent years. Note that the significant decrease in life expectancy at birth $e_0(t)$ for East German men in 1990 diminishes in the smoothed values $e_0(t)^S$.

For estimating tempo-adjusted life expectancy $e_0^*(t)$ the original values for $e_0(t)$ are substituted by the values $e_0(t)^S$ derived from the polynomial functions. To finally solve equation (2) for $e_0^*(t)$ I used the so-called Euler's method as described in its general form in appendix (a) of this paper with $S(1950) = 2$ as the initial condition for the differential equation. From equation (1) then follows that $e_0^*(1950)$ can be directly estimated from $e_0(1950) - S(1950)$. For instance for West German males follows $e_0^*(1950) = 71.28 - 2.00 = 69.28$. This value represents the assumed tempo distortion for mortality changes until the year 1950 that was equally set for all observed populations, thus the female and male populations of West and East Germany. It is important to note that the results for the analysed years after 1975 are insensitive to this assumed initial condition for the year 1950. Applying Euler's method leads to an estimate for the tempo-adjusted life expectancy $e_0^*(1951)$ for the next year from the equation

$$e_0^*(1951) = e_0^*(1950) + \left\{1 - \exp\left[-b(1950) \cdot \left(e_0(1950)^S - e_0^*(1950)\right)\right]\right\} ,$$

or generally written from

$$e_0^*(t+1) = e_0^*(t) + \left\{1 - \exp\left[-b(t) \cdot \left(e_0(t)^S - e_0^*(t)\right)\right]\right\} . \tag{3}$$

Equation (3) was used for estimating a complete time series of values for tempo-adjusted life expectancy at birth (with no mortality under age 30) until the year 2000 for the females and males of West and East Germany. The detailed derivation of equation (3) can be found in appendix (b).

Trends in tempo-adjusted life expectancy in West and East Germany

Figure 4 shows the trends in conventional and tempo-adjusted life expectancy at birth (both with no mortality under age 30) from 1975 to 2000 for females and males of West and East Germany. The graph for West German females (figure 4c) is very similar to the figures for US and Japanese women presented by Bongaarts and Feeney (2002: 24). As can be seen in figure 3(c), West German females are the only of the four analysed populations where the observed life expectancy at birth increased almost constantly since 1950. Thus, the tempo distortion $S(t)$ (what is defined as the difference between observed and tempo-adjusted life expectancy) is almost constant among West German females during the observation period from 1975 to 2000. Since among the other three populations improvements in life expectancy developed considerably later (West and East German males) or with significantly changing pace (East German females), tempo distortions must vary considerably as compared to West German females. This is perfectly reflected in the gained results for $e_0^*(t)$ and $S(t)$ as can clearly be seen in figure 4. In all cases the estimated tempo distortions agree perfectly with the logic of mortality tempo effects as described in the previous chapters.

This becomes especially clear by a comparison of figures 3 and 4. Among West German men the tempo distortion $S(t)$ was very low in 1975 and increased then steadily until the second half of the 1980s when the difference between observed and tempo-adjusted life expectancy reached an almost constant level (figure 4a). Since it becomes clear from figure 3(a) that life expectancy of West German men remained more or less unchanged between 1950 and 1970 there cannot be expected a noticeable tempo distortion in the mid 1970s. The increasing life expectancy after 1970 is caused a by shift in average age at death and consequently also the tempo-adjusted life expectancy starts to increase, but with a lower pace than observed life expectancy (what is absolutely consistent with the described cause of mortality tempo effects). Among East German males, life expectancy remained constant or even decreased slightly until the end of the 1980s before it started to increase with a much higher pace than in any phase of life expectancy trends in West Germany (figures 3a and 3b). Consequently, tempo-adjusted life expectancy $e_0^*(t)$ does not differ from unadjusted life expectancy until the beginning of the 1990s and then started to increase with a considerably lower speed as compared to $e_0(t)$. The annual increase in tempo-adjusted life expectancy after 1990 seems similar among West and East German males. Also among East German females the extent of tempo distortions in conventional life expectancy increased during the observed period. From figure 3(d) we know that life expectancy of East German females already rose before Reunification but with a lower pace than it did among their West German counterparts. Consequently, tempo distortions, i.e. the difference between tempo-adjusted and unadjusted life expectancy, remained at an almost constant level between 1975 and 1990. However, the difference between $e_0(t)$ and $e_0^*(t)$ started to increase at the end of the 1980s when conventional life expectancy rose with an increased speed similar to what can be observed among East German males (figures 4b and 4d).

The interesting question is how life expectancy differences between West and East Germany developed in the observation period 1975 to 2000 once adjusted for tempo distortions. The corresponding results are given in figure 5 for males and figure 6 for females, the single values can be found in table 2. The thinner lines in the two graphs represent the absolute difference between West and East Germany in conventional life expectancy and the bold lines those in tempo-adjusted life expectancy. Figure 5 depicts again the rapid decrease in conventional life expectancy differences after 1990 following an almost continuous increase since the beginning of the observation period. While West German males had a higher life expectancy according to conventional calculation methods since 1976, East German men showed a higher tempo-adjusted life expectancy until 1981. Note that the different definitions of live birth do not affect the results presented in this chapter and thus there seemed to be a "real" East German survival advantage among men in the 1970s. Only after 1981 the differences switched to an advantage of West German males, but to a much lower extend as compared to the results based on conventional life expectancy. The graph shows clearly that the increasing trend of West-East differences occurred with a considerable lower pace once life expectancy is adjusted for tempo effects. In 1990, when the difference in conventional life expectancy between West and East German men reached its peak of 3.08 years, the difference in tempo-adjusted life expectancy was nearly two years less with only 1.22 years. Even more interesting is the finding that the differences in tempo-adjusted life expectancy did not decrease since

Reunification but went on increasing until the end of the 1990s. While the differences in conventional life expectancy between West and East German males decreased to roughly one and a half years in 2000, the difference in tempo-adjusted life expectancy is now even higher with a difference of almost two years. Only in the second half of the 1990s the trend in increasing differences in tempo-adjusted life expectancy lowered its speed and indicates a convergence solely in the very last year of the observation period.

The results for West-East-German differences among females are similar to those just described for males. The survival advantage for West German females is considerably lower when tempo-adjusted life expectancy is used instead of conventional life expectancy. Since among females the trends in conventional life expectancy in West and East Germany before Reunification were not as different as among males, the relative trends of increasing West-East-differences in conventional and tempo-adjusted life expectancy are more similar as compared to the differences between West and East German males, i.e. the tempo distortions in the measured trend are less marked. However, this does not hold for the absolute differences in $e_0(t)$ and $e_0^*(t)$. While the differences in conventional life expectancy increased to 2.85 years in 1988, the differences in tempo-adjusted life expectancy did not exceed an extent of 2 years. Similar to the situation among men, the differences in tempo-adjusted life expectancy between West and East German females did not decrease with Reunification parallel to conventional life expectancy but increased until the mid 1990s. The trends in conventional and tempo-adjusted life expectancy crossed over between 1992 and 1993. From then on, the survival advantage of West German females measured with tempo-adjusted life expectancy is considerably higher as compared to the results based on tempo-unadjusted values. Although a decreasing trend in mortality differences between West and East German females can also be stated with tempo-adjusted life expectancy since the mid 1990s, the remaining differences in favour of West German women is still considerably higher. While the disadvantage of East German women decreased to 0.46 years in the year 2000 according to conventional life expectancy, the tempo-adjusted differences do still show an extent of 1.5 years.

Discussion

This paper presents both, a support of Bongaarts and Feeney's tempo approach in mortality and its first empirical application. If we accept the need for tempo-adjustment in the period

TFR we have equally to accept the need for tempo-adjustment in period life expectancy. The basic idea of the TFR is to estimate fertility quantum under current fertility conditions as a standardized indicator for current fertility conditions. Changes in the mean age at childbirth are causing tempo effects which affect age-specific fertility rates and thus the TFR that is based on them. Exactly the same holds for period life expectancy. The basic idea of life expectancy is to estimate the average length of life under current mortality conditions as a standardized indicator for current mortality conditions. Changes in the mean age at death are causing tempo effects which affect age-specific death rate and thus life expectancy that is based on them.

In principal, the adjustment formulas for the TFR and life expectancy follow the same basic idea. In the case of fertility it is assumed a shift of the age-specific fertility schedule, in the case of mortality a shift of the age-specific mortality schedule. Since, however, TFR and life expectancy are fundamentally different in their structural designs, also the adjustment formulas must include fundamental differences. The tempo-adjusted TFR for a certain period depends only on age-specific fertility rates in a small neighbourhood of the analyzed calendar years. This does not hold for the Bongaarts and Feeney formula for tempo-adjusted life expectancy. The decisive difference to the fertility procedure is that the proposed adjustment method for life expectancy uses a series of previous period life tables. Consequently, it is quite clear that the Bongaarts and Feeney formula reflects past mortality conditions in a certain way. But in the logic of tempo distortions this is not necessarily an inconsistency, especially not when the past changes in mortality conditions were steady and continuous what approximately holds for adult ages in developed populations and in the last decades. Since these are exactly the restrictions Bongaarts and Feeney (2002) give to their tempo-adjustment formula for life expectancy, we should not see a problem in the fact that it leads to values close (but not exactly) to a weighted moving average of past period life expectancy as Wachter (2005) has shown. In contrary, in restricting the application to industrialized countries of the recent past this property of the method is absolutely consistent with the theoretical idea of tempo distortions in life expectancy.

However, we cannot see the Bongaarts and Feeney formula as providing a perfect measure for tempo-adjusted period mortality conditions. This demand on the Bongaarts and Feeney method must already be excluded by noting that in practice all the assumptions will never be fulfilled. Thus, we should see the Bongaarts and Feeney formula as an attempt to standardize for tempo effects in period life expectancy in order to get a better measure for comparing period mortality conditions. It is, however, not clear to which extent the Bongaarts and Feeney method catches the real tempo effects and it is not possible to assess if it presents a maximum distortion in the sense that the truth lies somewhere between conventional and adjusted life expectancy. Nevertheless, it is important to separate this methodological aspects from the question of the general existence of tempo effects in period life expectancy.

The empirical results presented in the previous chapter are striking and could be of innermost importance for the general understanding of several phenomenon connected with changing mortality: once life expectancy is adjusted for tempo effects, differences between West and East Germany do not decrease immediately after Reunification and are ten years later considerably higher as compared to differences in conventional life expectancy. This indicates that the discussion about the reasons for the trends in mortality differences between West and East Germany of the last years might have been based on inappropriate measures and thus led to the wrong direction as already supposed by Luy (2004a). The puzzling aspect is that despite the active research of several scholars no factor can be found corresponding to the observed trends in conventional life expectancy at birth. However, according to the trends in West-East German differences in tempo-adjusted life expectancy, the explanatory factors do obviously not have to decrease the gap in life expectancy by more than 2 years among females respective 1.5 years among males within ten calendar years and they do not have to change mortality trends immediately after Reunification from one year to another. Research should rather be focused on finding factors producing an immediate and continuously increasing rise in average age at death that cause these tempo distortions in life expectancy but do not necessarily have to increase the average length of life to the extent indicated by conventional life expectancy. Obviously, the same holds for the phase of increasing differences prior to Reunification under changed conditions with higher tempo distortions in life expectancy for the West German population.

This paper presents a strong argument that the real differences in mortality conditions between West and East Germany differ considerably from what we thought they were. From this point of view it is not surprising that none of the usually stated explanatory variables for the West-East German survival gap fit to the observed mortality trends measured with conventional life expectancy at birth. To catch up the central message of Vaupel et al. derived from the closing West-East mortality gap in Germany (2003): it may in fact be never too late to increase the length of life, but it seems to take longer than trends in conventional life expectancy suggest, and the reasons might be of completely different kind than generally expected.

Moreover, since life expectancy without any adjustment for tempo effects is one of the most used demographic tools in order to analyze mortality we might have to change much more of what we thought to know on the base of this measure:

- What about the opening and the recent closing of the sex mortality gap in the developed world?
- What about the linear increase in record life expectancy at birth described by Oeppen and Vaupel (2002), especially regarding the impressive slope of this increase?
- What about the increasing mortality gap between Eastern and Western Europe?

While tempo-adjustments of the TFR lead mainly to somewhat higher estimates for the hypothetical family size under current fertility conditions this paper shows that tempo-adjustment of life expectancy can provide a completely different picture of current mortality conditions as compared to conventional life expectancy. We can expect that tempo effects distort the analysis in all cases where the compared populations experienced different trends in changing mortality. According to the simple illustrating example used at the beginning of this paper we should not doubt the existence of tempo effects in period life expectancy and the distortions they possibly cause. More critical might be the used method proposed by Bongaarts and Feeney (2002) since it is based on a number of assumptions that will never be completely satisfied. Accepting the existence of tempo effects, however, the used way of estimating tempo distortions should be preferred against using unadjusted estimates for period life expectancy as long as there are no better solutions. Thus, the main goal of formal demographer's future work should be the development of methods for tempo-adjusted life expectancy based on less restrictive assumptions that can be applied to all contemporary and past populations.

Acknowledgements

For methodological advice and for running specific computer programs I am very grateful to Paola Di Giulio and Roland Rau, as well as to James Vaupel for fruitful discussions and helpful comments on an earlier version of this paper that was presented at the IUSSP XXV International Population Conference in Tours, France, July 18-23, 2005.

References

Becker N., Boyle P., 1997: "Decline in mortality from testicular cancer in West Germany after reunification", *The Lancet* 350: 744.

Bobak M., Marmot M., 1996a: "East-west mortality divide and its potential explanations: proposed research agenda", *British Medical Journal* 312: 421-425.

Bobak M., Marmot M., 1996b: "East-west health divide and potential explanations", in: Hertzman C., Kelly S., Bobak M. (Ed.): *East-west life expectancy gap in Europe. Environmental and non-environmental determinants*, Dordrecht et al.: Kluwer: 17-44.

Bongaarts J., Feeney G., 1998: "On the quantum and tempo of fertility", *Population and Development Review* 24: 271-291.

Bongaarts J., Feeney G., 2002: "How long do we live?", *Population and Development Review* 28: 13-29.

Bongaarts J., Feeney G., 2003: "Estimating mean lifetime", *Proceedings of the National Academy of Science* 100: 13127-13133.

Bourgeois-Pichat J., 1985: "Recent changes in mortality in industrialized countries", in: Vallin J., Lopez A. D., Behm H. (Ed.): *Health policy, social policy and mortality prospects*, Liege: Ordina Editions: 507-539.

Boys R. J., Forster D. P., Józan P., 1991: "Mortality from causes amenable and non-amenable to medical care: the experience of eastern Europe", *British Medical Journal* 303: 879-883.

Brouard N., 1982: "Structure et dynamique des populations. La pyramide des années à vivre, aspects nationaux et examples régionaux", *Espaces, Populations, Sociétés* 2: 157-168.

Bucher H., 2002: "Die Sterblichkeit in den Regionen der Bundesrepublik Deutschland und deren Ost-West-Lücke seit der Einigung", in: Cromm J., Scholz R. D. (Ed.): *Regionale Sterblichkeit in Deutschland*, Augsburg and Göttingen: WiSoMed, Cromm: 33-38.

Caselli G., Egidi V., 1980: *New trends in European mortality*, Strasbourg: Council of Europe.

Chruscz D., 1992: "Zur Entwicklung der Sterblichkeit in geeinten Deutschland: die kurze Dauer des Ost-West-Gefälles", *Informationen zur Raumentwicklung* 9-10: 691-700.

Dinkel R. H., 1992: "Kohortensterbetafeln für die Geburtsjahrgänge 1900 bis 1962 in den beiden Teilen Deutschlands", *Zeitschrift für Bevölkerungswissenschaft* 18: 95-116.

Dinkel R. H., 1994: "Die Sterblichkeitsentwicklung der Geburtsjahrgänge in den beiden deutschen Staaten. Ergebnisse und mögliche Erklärungshypothesen", in: Imhof A. E., Weinknecht R. (Ed.): *Erfüllt leben – in Gelassenheit sterben: Geschichte und Gegenwart*, Berlin: Duncker & Humblot: 155-170.

Dinkel R. H., 1999: *East and West German mortality before and after reunification*, unpublished manuscript, University of Rostock.

Dorbritz J., Gärtner K., 1995: "Bericht 1995 über die demographische Lage in Deutschland", *Zeitschrift für Bevölkerungswissenschaft* 20: 339-448.

Eberstadt N., 1994: "Demographic shocks after Communism: Eastern Germany, 1989-93", *Population and Development Review* 20: 137-152.

Feeney G., 2003: *Mortality tempo: a guide for the sceptic*, unpublished manuscript, to be downloaded from http://www.gfeeney.com.

Gjonça A., Brockmann H., Maier H., 2000: "Old-age mortality in Germany prior to and after Reunification", *Demographic Research* 3: Article 1.

Guillot M., 2003a: "The cross-sectional average length of life (CAL): a cross-sectional mortality measure that reflects the experience of cohorts", *Population Studies* 57: 41-54.

Guillot M., 2003b: "Does period life expectancy overestimate current survival? An analysis of tempo effects in mortality", paper presented at the *PAA 2003 Annual Meeting*, Minneapolis, Minnesota, USA, May 1-3.

Häussler B., Hempel E., Reschke P., 1995: "Die Entwicklung von Lebenserwartung und Sterblichkeit in Ostdeutschland nach der Wende (1989-1992)", *Öffentliches Gesundheitswesen* 57: 365-372.

Heinemann L., Dinkel R. H., Görtler E., 1996: "Life expectancy in Germany: possible reasons for the increasing gap between East and West Germany", *Reviews on Environmental Health* 11: 15-26.

Hertzman C., Kelly S., Bobak M. (Ed.), 1996: *East-west life expectancy gap in Europe. Environmental and non-environmental determinants*, Dordrecht et al.: Kluwer.

Hobcraft J., 2004: "Method, theory, and substance in understanding choices about becoming a parent: Progress or regress? Discussion of the paper by Caldwell and Schindlmayr", *Population Studies* 58: 81-84.

Höhn C., Pollard J., 1990: "Analyse der Sterblichkeit in beiden Teilen Deutschlands in den Jahren 1976 bis 1986", *Zeitschrift für Bevölkerungswissenschaft* 16: 355-381.

Höhn C., Pollard J., 1991: "Mortality in the two Germanies in 1986 and trends 1976-1986", *European Journal of Population* 7: 1-28.

Kohler H.-P., Philipov D., 2001: „Variance effects in the Bongaarts-Feeney formula", *Demography* 38: 1-16.

Luy M., 2004a: “Mortality differences between Western and Eastern Germany before and after Reunification: a macro and micro level analysis of developments and responsible factors”, *Genus* 60: 99-141.

Luy M., 2004b: “Verschiedene Aspekte der Sterblichkeitsentwicklung in Deutschland von 1950 bis 2000”, *Zeitschrift für Bevölkerungswissenschaft* 29: 3-62.

Luy M., 2005: “Ursachen der Sterblichkeitsunterschiede zwischen West- und Ostdeutschland unter besonderer Berücksichtigung des Einflusses von Lebensstil und Lebensqualität”, in: Gärtner K., Grünheid E., Luy M. (Ed.): *Lebensstile, Lebensphasen, Lebensqualität – Interdisziplinäre Analysen von Gesundheit und Sterblichkeit aus dem Lebenserwartungssurvey des BiB*, Wiesbaden: VS-Verlag für Sozialwissenschaften, in preparation.

Luy M., Caselli G., 2004: “The impact of a migration-caused selection effect on regional mortality differences in Italy and Germany”, paper presented at the *PAA 2004 Annual Meeting*, Boston, Massachusetts, USA, April 1-3.

Mackenbach J., Velkova A., Van Den Bosch J., 1996: “The contribution of differences in mortality from conditions ammenable to medical intervention”, in: Hertzman C., Kelly S., Bobak M. (Ed.): *East-west life expectancy gap in Europe. Environmental and non-environmental determinants*, Dordrecht et al.: Kluwer: 61-69.

Mai R., 2004: “Regionale Sterblichkeitsunterschiede in Ostdeutschland. Struktur, Entwicklung und die Ost-West-Lücke seit der Wiedervereinigung”, in: Scholz R., Flöthmann J. (Ed.): *Lebenserwartung und Mortalität*, Materialien zur Bevölkerungswissenschaft 111, Wiesbaden: BiB: 51-68.

McKee M., Nolte E., 2004: “Health sector reforms in central and eastern Europe: how well are health services responding to changing patterns of health?”, *Demographic Research*, special collection 2, article 7: 163-181.

Meslé F., Hertrich V., 1997: “Mortality trends in Europe: the widening gap between east and west”, in: *23rd International Population Conference*, Beijing 1997, Liège: IUSSP: 479-508.

Meslé F., Vallin J., 2002: “Mortality in Europe: the divergence between east and west”, *Population*-E 57: 157-198.

Müller C. K. E., 1976: *Die Säuglingssterblichkeit in der Bundesrepublik Deutschland und in der Deutschen Demokratischen Republik*, Bonn: Schwarzbold.

Nolte E., Shkolnikov V., McKee M., 2000a: “Changing mortality patterns in east and west Germany and Poland: I. Long-term trends”, *Journal of Epidemiology and Community Health* 54: 890-899.

Nolte E., Shkolnikov V., McKee M., 2000b: “Changing mortality patterns in east and west Germany and Poland: II. Short-term trends during transition and in the 1990s”, *Journal of Epidemiology and Community Health* 54: 899-906.

Nolte E., Scholz R., Shkolnikov V., McKee M., 2002: “The contribution of medical care to changing life expectancy in Germany and Poland”, *Social Science & Medicine* 55: 1905-1921.

Nolte E., Scholz R., McKee M., 2004: “Progress in health care, progress in health? Patterns of amenable mortality in central and eastern Europe before and after political transition”, *Demographic Research*, special collection 2, article 6: 139-162.

Philipov D., Kohler H.-P., 2001: „Tempo effects in the fertility decline in Eastern Europe: evidence from Bulgaria, the Czech Republic, Hungary, Poland, and Russia“, *European Journal of Population* 17: 37-60.

Riphahn R. T., 1999: “Die Mortalitätskrise in Ostdeutschland und ihre Reflektion in der Todesursachenstatistik”, *Zeitschrift für Bevölkerungswissenschaft* 24: 329-363.

Rutstein D. D., Berenberg W., Chalmers T. C., Child C. G., Fishman A. P., Perrin E. B., 1976: “Measuring the quality of medical care. A clinical method”, *New England Journal of Medicine* 294: 582-588.

Scholz R. D., 1996: “Analyse und Prognose der Mortalitätsentwicklung in den alten und neuen Bundesländern – Ergebnisse des Ost/West-Vergleiches der Kohortensterblichkeit”, in: Dinkel R. H., Höhn C., Scholz R. D. (Ed.): *Sterblichkeitsentwicklung – unter besonderer Berücksichtigung des Kohortenansatzes*, München: Boldt: 89-102.

Schott J., Wiesner G., Casper W., Bergmann K. E., 1994: “Entwicklung der Mortalität des alten Menschen in Ost- und Westdeutschland in den zurückliegenden Jahrzehnten”, in: Imhof A. E., Weinknecht R. (Ed.): *Erfüllt leben – in Gelassenheit sterben: Geschichte und Gegenwart*, Berlin: Duncker & Humblot: 171-182.

Sobotka T., 2004: “Is lowest-low fertility in Europe explained by the postponement of childbearing?”, *Population and Development Review* 30: 195-220.

Statistisches Bundesamt, 2001: *Gesundheitswesen, Fachserie 12, Reihe 6.1: Grunddaten der Krankenhäuser und Vorsorge- oder Rehabilitationseinrichtungen*, Stuttgart: Metzler-Poeschel.

Vallin J., Meslé F., 2001: “Trends in mortality in Europe since 1950: age-, sex- and cause-specific mortality”, in: *Trends in mortality and differential mortality*, Strasbourg, Council of Europe Publishing (Population Studies No. 36): 31-186.

Vaupel J. W., 2002: “Life expectancy at current rates vs. current conditions: a reflexion stimulated by Bongaarts and Feeney’s ‘How Long Do We Live?’”, *Demographic Research* 7: 365-377.

Vaupel J. W., Carey J. R., Christensen K., 2003: “It’s never too late”, *Science* 301: 1679-1681.

Velkova A., Wolleswinkel-van den Bosch J. H., Mackenbach J. P., 1997: “The east-west life expectancy gap: differences from conditions amenable to medical intervention”, *International Journal of Epidemiology* 26: 75-84.

Wilmoth J. R., 2005: “On the relationship between period and cohort mortality”, paper presented at the *PAA 2005 Annual Meeting*, Philadelphia, Pennsylvania, USA, March 31 – April 2.

Winston F. K., Rineer C., Menon R., et al., 1999: “The carnage wrought by major economic change: ecological study of traffic related mortality and the reunification of Germany”, *British Medical Journal* 318: 1647-1649.

Zeng Yi, Land K. C., 2001: „A sensitivity analysis of the Bongaarts-Feeney method for adjusting bias in observed period total fertility rates“, *Demography* 38: 17-28.

Zeng Yi, Land K. C., 2002: “Adjusting period tempo changes with an extension of Ryder’s basic translation equation”, *Demography* 39: 269-285.

Appendix

(a) Euler's method to solve a differential equation

The following description of the so-called Euler's method is taken from "Keshet's Typed Notes (2004)", chapter 9.13, pp. 16-17.[1] In cases where it is difficult or impossible to find the desired solution for a differential equation with guesses, integration methods, or from previous experience, it is possible to use approximation methods and numerical computations to do the job. Most of these methods rely on the fact that derivatives can be approximated by finite differences. For example, suppose a differential equation of the form

$$\frac{dy}{dt} = f(y) \tag{4}$$

with initial value $y(0) = y_0$, can be approximated by selecting a set of time points t_1, t_2, …, which are spaced apart by time steps of size Δt, and replacing the differential equation by the approximate finite difference equation

$$\frac{y_1 - y_0}{\Delta t} = f(y_0). \tag{5}$$

This relies on the approximation

$$\frac{dy}{dt} \approx \frac{\Delta y}{\Delta t}, \tag{6}$$

which is a relatively good approximation for small step size Δt. Then by rearranging this approximation follows that

$$y_1 = y_0 + f(y_0)\Delta t. \tag{7}$$

Knowing the quantities on the right allows us to compute the value y_1, i.e. the value of the approximate "solution" at the time point t_1. It is then possible to continue to generate the value

[1] Keshet's textbook "Math 102 Course Notes (2004)" is online available and can be downloaded from http://ugrad.math.ubc.ca/coursedoc/math102/keshet.notes/index.html

at the next time point in the same way, by approximating the derivative again as a secant slope. This leads to

$$y_2 = y_1 + f(y_1)\Delta t \,. \tag{8}$$

The approximation generated in this way, leading to y_1, y_2, … is called Euler's method. The fact that this procedure is only an approximate solution for a differential equation should not lower the value of the results presented in this paper since the general Bongaarts/Feeney-equation (2) is based on more severe assumptions that are never perfectly satisfied. Thus, the results presented here are anyway expected to provide solely an impression of the extent to which tempo distortions might lead in the wrong direction in the case of analysing mortality differences between West and East Germany using conventional life table techniques. An important goal of future works in formal demography must be the development of new ways for estimating tempo-adjusted life expectancy based on less restrictive assumptions.

(b) Derivation of equation (3)

In order to apply Euler's method to solve equation (2) for $e_0^*(t)$, $f(y)\Delta t$ in equations (7) and (8) has to be substituted by $\{-1/b \cdot[1- de_0^*(t)/dt]\}$. From equations (1) and (2) and using annual estimates for the Gompertz parameter b follows that the tempo distortion $S(t)$ can be estimated by

$$S(t) = -\frac{1}{b(t)} \cdot \ln\left[1 - \frac{de_0^*(t)}{dt}\right] \,. \tag{9}$$

Since $b(t)$ varies only slightly over time (see tab. 1 and app. c and d), the annual estimates of $b(t)$ may be substituted by an average value for $b(t) = b$, as Bongaarts and Feeney (2002) proposed in their first paper on mortality tempo. However, since this would imply another additional assumption, I used the annual estimates $b(t)$. Equation (9) can be rearranged to

$$\ln\left[1 - \frac{de_0^*(t)}{dt}\right] = -b(t) \cdot S(t) \,, \tag{10}$$

what leads to

$$1-\frac{de_0^*(t)}{dt} = \exp\left[-b(t)\cdot S(t)\right]. \tag{11}$$

Since time is measured in units of annual steps and applying Euler's method (assuming that the change is linear), $de_0^*(t)/dt$ can be represented by $e_0^*(t) - e_0^*(t+1)$ and equation (11) becomes to

$$1-\left[e_0^*(t) - e_0^*(t+1)\right] = \exp\left[-b(t)\cdot S(t)\right], \tag{12}$$

what yields

$$e_0^*(t) - e_0^*(t+1) = 1-\exp\left[-b(t)\cdot S(t)\right] , \tag{13}$$

and thus

$$e_0^*(t+1) = e_0^*(t) + \left\{1-\exp\left[-b(t)\cdot S(t)\right]\right\} . \tag{14}$$

Using equation (1) and substituting $e_0(t)$ by $e_0(t)^S$ leads directly to equation (3). As described in the text, for the first year 1950 I used $S(1950) = 2.00$ as initial condition leading to an estimate of $e_0^*(1951)$ by using equation (3). The same procedure was then repeated for each subsequent calendar year in order to determine the complete series of estimates for $e_0^*(t)$ until the year 2000.

(c) Estimates of $e_0(t)$, $e_0(t)^S$, b, $e_0{}^*(t)$, and S(t) for single calendar years, males of West and East Germany, 1975-2000 (no mortality under age 30)

Year	West Germany					East Germany				
t	$e_0(t)$	$e_0(t)^S$	b	$e_0{}^*(t)$	$S(t)$	$e_0(t)$	$e_0(\mathrm{t})^S$	b	$e_0{}^*(t)$	$S(t)$
1975	71.17	71.41	0.107	70.97	0.200	71.28	71.41	0.114	71.41	-0.132
1976	71.49	71.54	0.106	71.02	0.468	71.46	71.41	0.110	71.41	0.052
1977	71.96	71.70	0.106	71.07	0.888	71.59	71.40	0.110	71.41	0.188
1978	71.85	71.87	0.107	71.14	0.710	71.33	71.39	0.111	71.41	-0.076
1979	72.20	72.05	0.105	71.21	0.987	71.20	71.38	0.112	71.40	-0.208
1980	72.29	72.25	0.104	71.30	0.997	71.07	71.38	0.113	71.40	-0.328
1981	72.39	72.46	0.105	71.39	0.995	71.38	71.38	0.109	71.40	-0.023
1982	72.63	72.67	0.106	71.50	1.133	71.44	71.38	0.109	71.40	0.041
1983	72.78	72.89	0.106	71.62	1.160	71.66	71.38	0.109	71.39	0.270
1984	73.18	73.11	0.107	71.74	1.441	71.70	71.39	0.111	71.39	0.308
1985	73.26	73.32	0.108	71.88	1.378	71.57	71.41	0.112	71.39	0.173
1986	73.54	73.54	0.107	72.02	1.516	71.51	71.44	0.111	71.39	0.118
1987	73.84	73.74	0.110	72.17	1.662	71.70	71.48	0.110	71.40	0.301
1988	74.07	73.93	0.109	72.33	1.736	71.56	71.53	0.109	71.41	0.153
1989	74.13	74.12	0.111	72.49	1.640	71.88	71.61	0.109	71.42	0.456
1990	74.22	74.29	0.110	72.66	1.560	71.14	71.70	0.106	71.44	-0.302
1991	74.35	74.46	0.108	72.82	1.526	71.31	71.83	0.103	71.47	-0.160
1992	74.68	74.62	0.108	72.98	1.692	71.77	71.99	0.107	71.50	0.267
1993	74.70	74.78	0.110	73.15	1.551	72.01	72.19	0.108	71.55	0.457
1994	74.96	74.95	0.111	73.31	1.644	72.40	72.44	0.106	71.62	0.780
1995	75.05	75.13	0.109	73.48	1.569	72.81	72.74	0.111	71.70	1.108
1996	75.27	75.33	0.110	73.64	1.630	73.27	73.11	0.109	71.81	1.459
1997	75.68	75.56	0.111	73.81	1.870	73.89	73.55	0.112	71.94	1.947
1998	75.99	75.85	0.110	73.99	2.004	74.43	74.09	0.112	72.11	2.324
1999	76.28	76.22	0.111	74.17	2.102	74.82	74.72	0.115	72.31	2.513
2000	76.54	76.67	0.109	74.38	2.165	75.03	75.46	0.113	72.55	2.477

(d) Estimates of $e_0(t)$, $e_0(t)^S$, b, $e_0^*(t)$, and $S(t)$ for single calendar years, females of West and East Germany, 1975-2000 (no mortality under age 30)

Year	West Germany					East Germany				
t	$e_0(t)$	$e_0(t)^S$	b	$e_0^*(t)$	$S(t)$	$e_0(t)$	$e_0(t)^S$	b	$e_0^*(t)$	$S(t)$
1975	76.84	77.04	0.118	75.79	1.048	75.91	76.05	0.123	75.47	0.436
1976	77.17	77.23	0.116	75.93	1.242	76.15	76.09	0.121	75.54	0.607
1977	77.74	77.44	0.119	76.07	1.668	76.53	76.13	0.120	75.61	0.927
1978	77.74	77.66	0.117	76.22	1.524	76.34	76.18	0.123	75.67	0.669
1979	78.03	77.89	0.116	76.37	1.657	76.40	76.24	0.123	75.73	0.676
1980	78.27	78.13	0.119	76.54	1.735	76.17	76.30	0.122	75.79	0.384
1981	78.34	78.37	0.118	76.71	1.636	76.37	76.37	0.121	75.85	0.517
1982	78.57	78.62	0.119	76.89	1.684	76.57	76.46	0.121	75.91	0.663
1983	78.76	78.86	0.119	77.07	1.685	76.82	76.56	0.123	75.97	0.840
1984	79.23	79.10	0.119	77.26	1.969	76.85	76.67	0.123	76.04	0.804
1985	79.30	79.32	0.121	77.46	1.844	76.77	76.80	0.124	76.12	0.651
1986	79.47	79.54	0.122	77.66	1.812	76.73	76.95	0.128	76.20	0.527
1987	79.82	79.74	0.124	77.87	1.951	77.14	77.13	0.127	76.29	0.852
1988	80.02	79.93	0.121	78.07	1.947	77.17	77.32	0.124	76.39	0.777
1989	80.07	80.10	0.121	78.27	1.794	77.49	77.54	0.121	76.50	0.987
1990	80.05	80.25	0.122	78.47	1.581	77.44	77.79	0.120	76.62	0.824
1991	80.26	80.39	0.123	78.67	1.588	77.69	78.06	0.118	76.75	0.939
1992	80.59	80.52	0.121	78.86	1.734	78.38	78.36	0.124	76.89	1.486
1993	80.50	80.63	0.122	79.04	1.465	78.86	78.68	0.122	77.06	1.800
1994	80.78	80.75	0.123	79.22	1.560	79.18	79.03	0.135	77.24	1.936
1995	80.89	80.87	0.123	79.39	1.501	79.53	79.41	0.125	77.45	2.080
1996	80.98	81.01	0.122	79.56	1.424	79.86	79.82	0.122	77.67	2.190
1997	81.32	81.18	0.125	79.72	1.606	80.43	80.25	0.131	77.90	2.527
1998	81.55	81.40	0.125	79.88	1.665	80.76	80.70	0.128	78.17	2.590
1999	81.71	81.69	0.125	80.06	1.653	81.18	81.17	0.131	78.44	2.739
2000	81.93	82.08	0.125	80.24	1.687	81.47	81.66	0.130	78.74	2.721

Fig. 1: Mortality tempo effect illustrated in the Lexis diagram

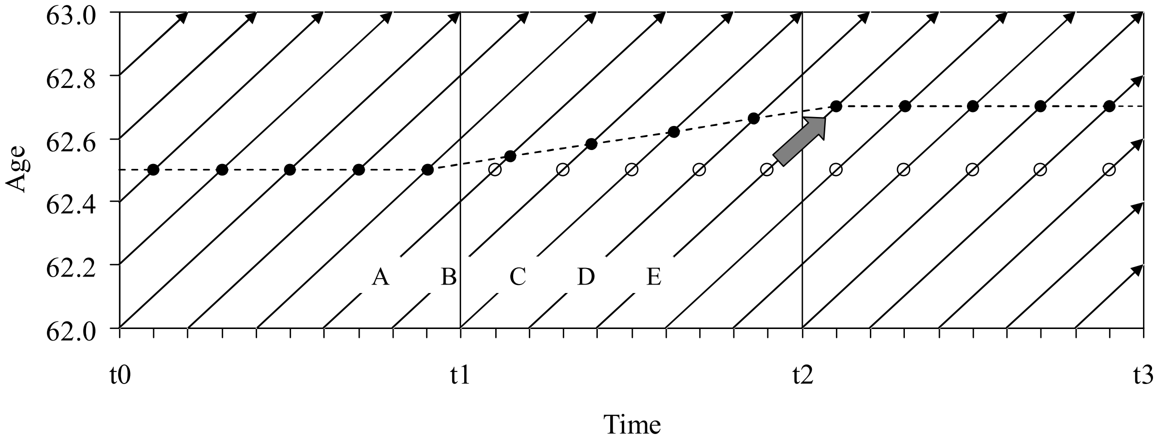


Fig. 2: Trends in life expectancy at birth for West and East Germany, 1950-2000

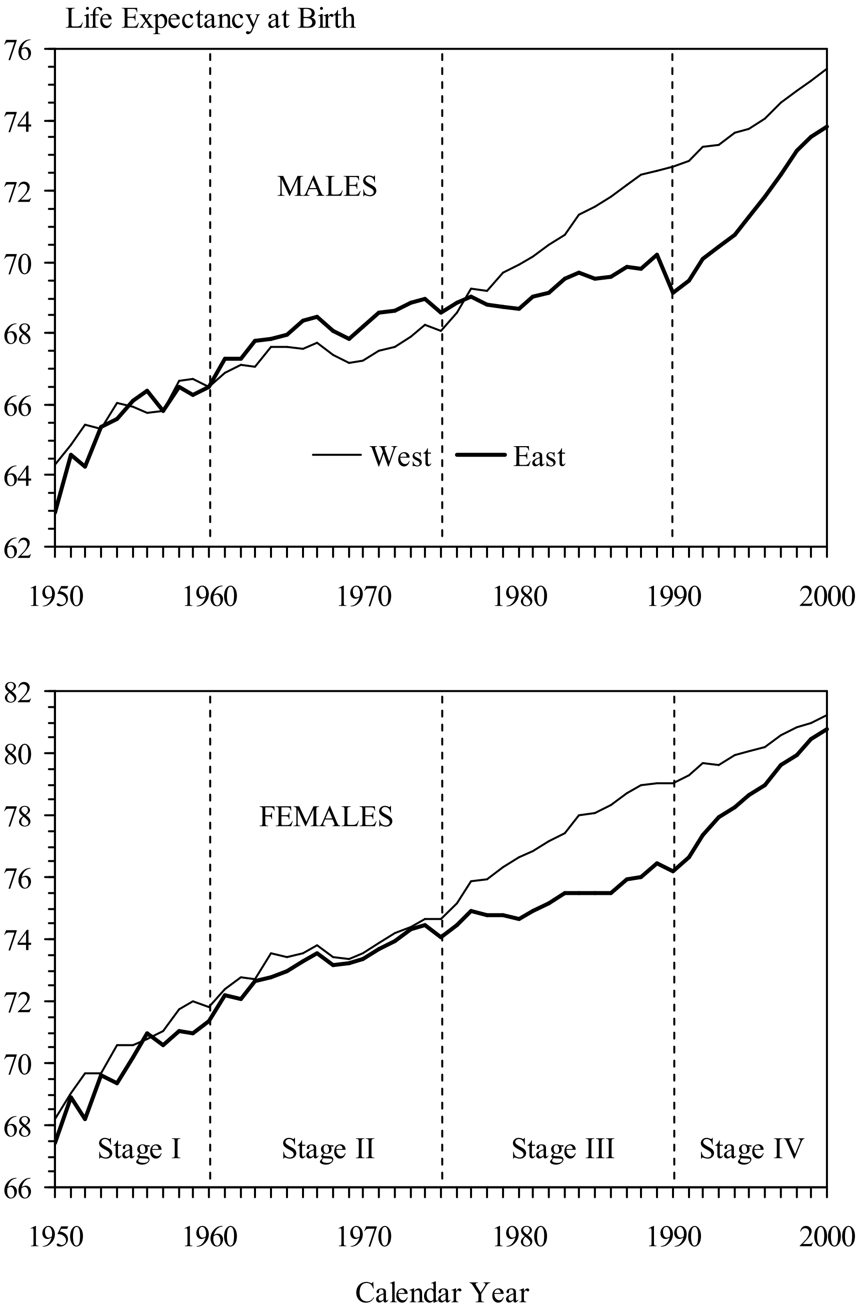


Fig. 3: Observed life expectancy at birth $e_0(t)$ and smoothed estimates $e_0(t)^S$ (sixth degree polynomial), West and East Germany, 1950-2000 (no mortality under age 30)

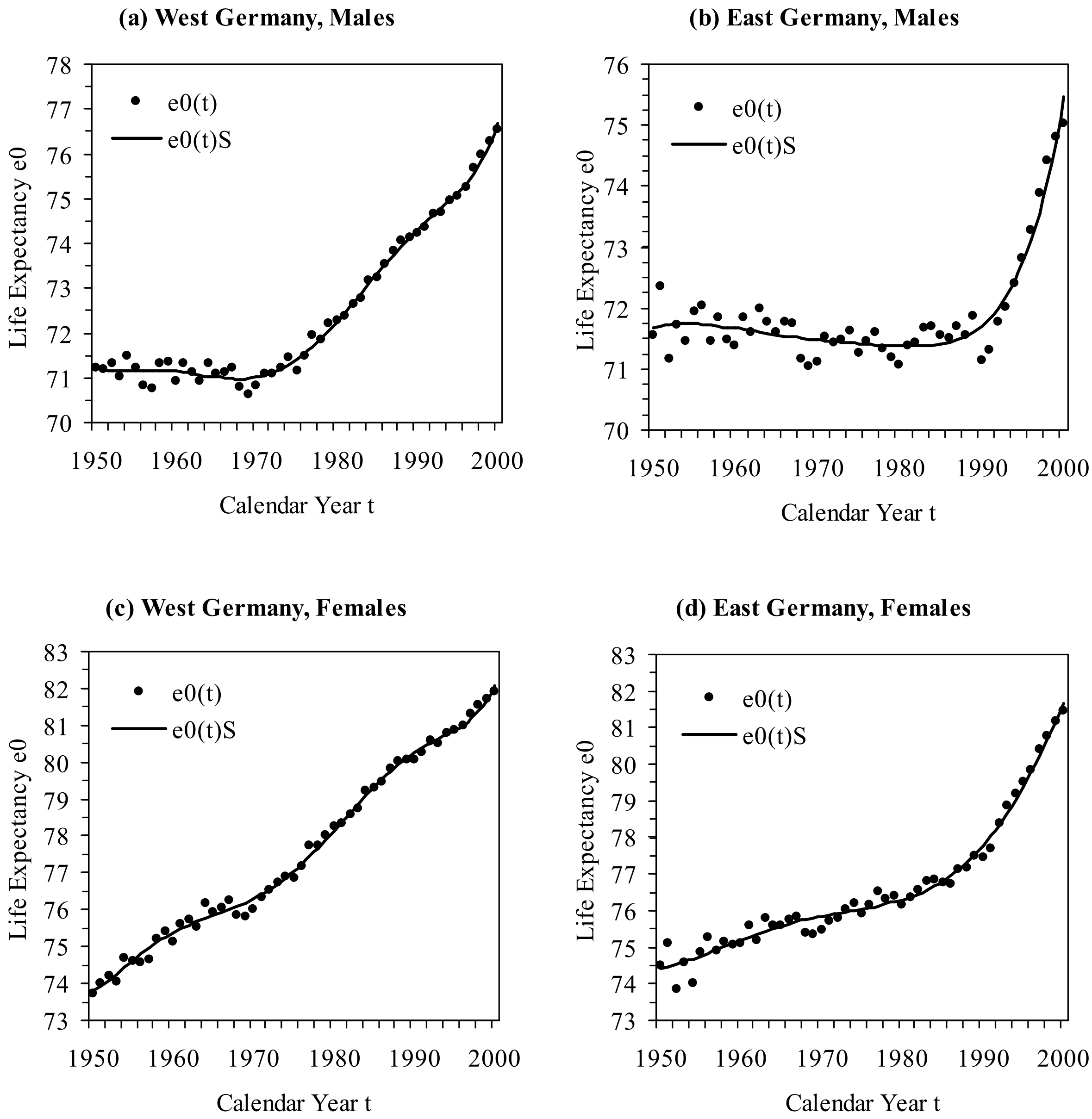


Fig. 4: Observed life expectancy at birth $e_0(t)$ and estimated mean age at death $e_0^*(t)$ (adjusted life expectancy at birth) with tempo distortion S(t), West and East Germany, 1975-2000 (no mortality under age 30)

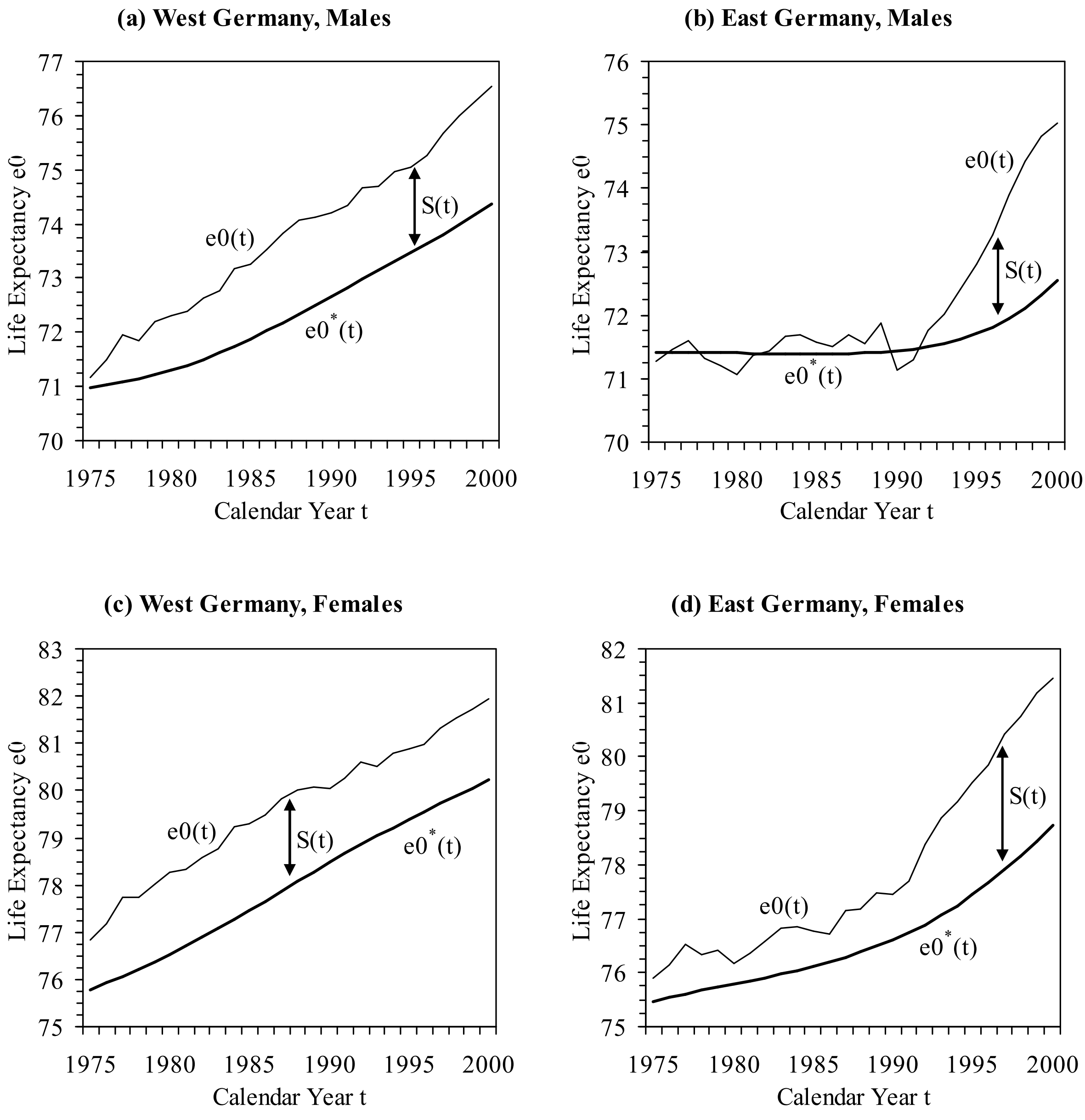


Fig. 5: West-East German difference in life expectancy at birth for conventional life expectancy $e_0(t)$ and tempo-adjusted life expectancy $e_0^*(t)$, Males 1975-2000 (no mortality under age 30)

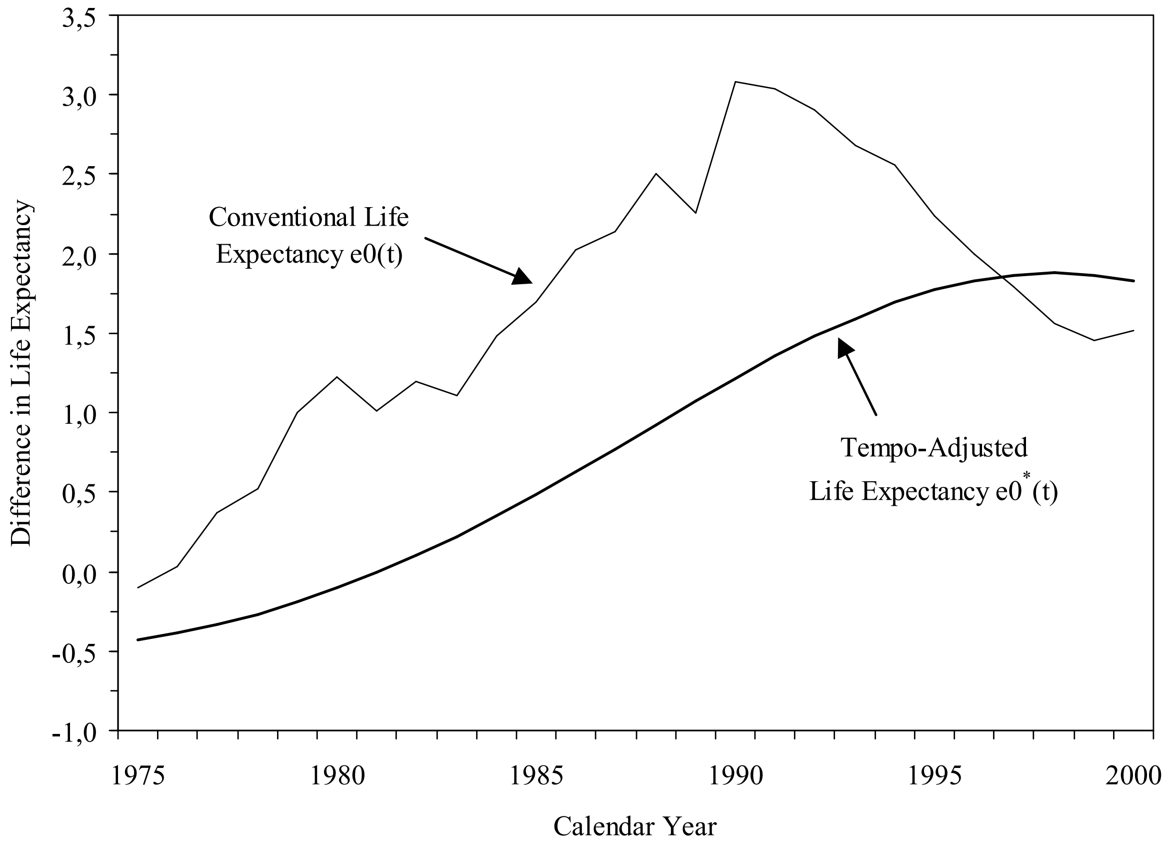
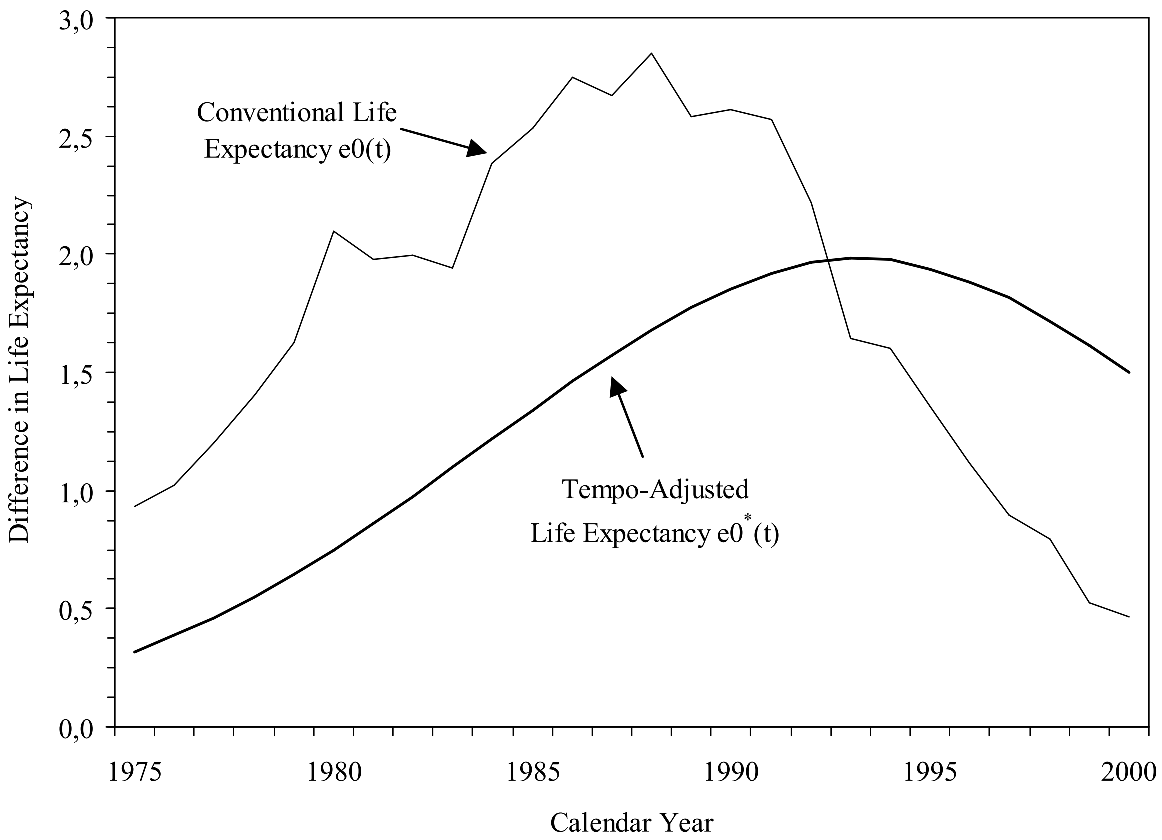


Fig. 6: West-East German difference in life expectancy at birth for conventional life expectancy $e_0(t)$ and tempo-adjusted life expectancy $e_0^*(t)$, Females 1975-2000 (no mortality under age 30)



Tab. 1: Estimates of parameters of the Gompertz mortality change model for males and females in West and East Germany, 1975-2000

			Average 1975-2000		
	μ_0(1975)	μ_0(2000)	b	St. dev. b	R^2
West Germany, Males	6.161 ($\cdot 10^{-5}$)	3.142 ($\cdot 10^{-5}$)	0.108	0.0020	0.991
East Germany, Males	4.737 ($\cdot 10^{-5}$)	3.568 ($\cdot 10^{-5}$)	0.110	0.0026	0.993
West Germany, Females	2.206 ($\cdot 10^{-5}$)	7.644 ($\cdot 10^{-6}$)	0.121	0.0028	0.993
East Germany, Females	1.591 ($\cdot 10^{-5}$)	7.483 ($\cdot 10^{-6}$)	0.124	0.0040	0.995

Tab. 2: West-East German difference in life expectancy at birth according to conventional $e_0(t)$ and tempo-adjusted $e_0^*(t)$, 1975-2000 (no mortality under age 30)

Years Before Reunification					Years After Reunification				
	Males		Females			Males		Females	
Year	$e_0(t)$	$e_0^*(t)$	$e_0(t)$	$e_0^*(t)$	Year	$e_0(t)$	$e_0^*(t)$	$e_0(t)$	$e_0^*(t)$
1975	-0.10	-0.43	0.93	0.32	1990	3.08	1.22	2.61	1.85
1976	0.03	-0.39	1.02	0.39	1991	3.04	1.35	2.57	1.92
1977	0.37	-0.33	1.20	0.46	1992	2.90	1.48	2.21	1.97
1978	0.52	-0.27	1.41	0.55	1993	2.69	1.59	1.65	1.98
1979	1.00	-0.19	1.63	0.65	1994	2.55	1.69	1.60	1.98
1980	1.22	-0.10	2.10	0.75	1995	2.23	1.77	1.36	1.93
1981	1.01	-0.01	1.98	0.86	1996	2.00	1.83	1.12	1.88
1982	1.19	0.10	2.00	0.98	1997	1.79	1.87	0.90	1.82
1983	1.11	0.22	1.94	1.10	1998	1.56	1.88	0.79	1.72
1984	1.48	0.35	2.38	1.22	1999	1.46	1.87	0.53	1.61
1985	1.69	0.49	2.53	1.34	2000	1.52	1.83	0.46	1.50
1986	2.03	0.63	2.75	1.46					
1987	2.13	0.77	2.67	1.57					
1988	2.51	0.92	2.85	1.68					
1989	2.25	1.07	2.58	1.77					

Endnotes

[1] In the mentioned paper Vaupel (2002) connected the distortions inherent in current mortality rates as a consequence of a changed timing of death rather with effects of heterogeneity than with effects of mortality tempo. Regardless the specific sight of the origin of distortions in period mortality rates Vaupel`s message is universe for all kinds of demographic period measures.

[2] In West Germany, the result of a delivery is defined as a live birth if one of the three signs of life, namely heart-beat, natural respiration, or pulsating umbilical cord, is recognised. According to East German statistics, a live birth was defined only by the joint existence of heartbeat and natural respiration (Müller, 1976). Consequently, all deaths of new-borns showing only one of the three signs of life are registered as live births, and thus as infant deaths only in West Germany, while in East Germany such cases were registered as stillbirths, and did not enter infant mortality statistics.

[3] The application of a Gompertz model requires the assumption that mortality under age 30 is negligible since the Gompertz model does not fit to the pattern of mortality in ages below 30. Since this assumption is close to reality in modern populations with high life expectancy it can be accepted as sufficiently satisfied in the case of West and East Germany from 1975 to 2000. However, this method cannot be used in populations with high mortality in infancy as well as in childhood and young adult ages.

[4] Bongaarts and Feeney (2002) fitted the age-specific death rates until age 100, but in the case of West and East Germany official population and death data is only available until age 90.